

KUWAIT UNIVERSITY
Department of Mathematics and Computer Science

Math 101
Calculus (1)

Second Midterm

10 May, 2009
Time: 90 minutes

Answer all questions. Calculators and Mobile Phones are not allowed.

1. (3pts) Let $f(t) = t^2 \sin t$. Find $f''(0)$.
2. (3pts) Suppose $y = f(x)$ is given by $x^2 + y^2 = 2y$. Find the critical numbers of f .
3. (3pts) A car starts at a point A and travels east at 80km/hr. At the same time another car starts at A and goes north at 60km/hr. At what rate is the distance between them changing one hour after the cars start?
4. (3pts) Find the linear approximation of $f(x) = \sqrt{2x+7}$ at $a = 1$.
5. (3pts) Let $f(x) = 6(2-x)^{4/3} + 5$.
 - (a) Show that there is no $c \in (1, 2)$ such that $f'(c) = 0$.
 - (b) Does this contradict Rolle's Theorem for f on $[1, 2]$? Explain.
6. (10pts) Let $f(x) = x^4 - 6x^2 + 2$.
 - (a) Discuss the symmetry of the graph of $y = f(x)$.
 - (b) Find the intervals on which f is increasing and the intervals on which f is decreasing. Find the local extrema of f , if any.
 - (c) Find the intervals on which f is concave upwards and the intervals on which f is concave downwards. Find the inflection points of f , if any.
 - (d) Find the absolute extrema of f on $[-2, 3]$.
 - (e) Sketch the graph of f .

Solutions

1. $f' = 2t \sin t + t^2 \cos t$ (1pt), $f'' = 2 \sin t + 4t \cos t - t^2 \sin t$ (1pt), so $f''(0) = 0$ (1pt).

2. $2x + 2yy' = 2y'$, so $y' = \frac{x}{1-y}$ (1pt). Now, $y' = 0 \iff x = 0$ and does not exist
 $\iff y = 1, x = \pm 1$ (1pt). Critical points are $0, \pm 1$ (1pt).

3. Let x be distance travelled by first car, y distance travelled by second car after t hours. Distance between them is

$D = \sqrt{x^2 + y^2}$ (1pt), $\frac{dD}{dt} = \frac{2xx' + 2yy'}{2\sqrt{x^2 + y^2}}$ (1pt), so when $t = 1, x = 80, y = 60, x' = 80, y' = 60, dD/dt = 100$ (1pt).

4. $f' = \frac{2}{2\sqrt{2x+7}}$ (1pt), so $L = f(a) + f'(a)(x - a)$ (1pt) $= 3 + \frac{1}{3}(x - 1)$ (1pt).

5. $f' = -6\frac{4}{3}(2 - x)^{1/3}$ (1pt) $= 0 \iff x = 2$, so no $c \in (1, 2)$ (1pt). Notice that $f(1) = 11 \neq f(2) = 5$, so hypotheses of Rolle not satisfied(1pt).

6. (a) (1pt) $f(-x) = f(x)$, f even, graph symmetric about y -axis.

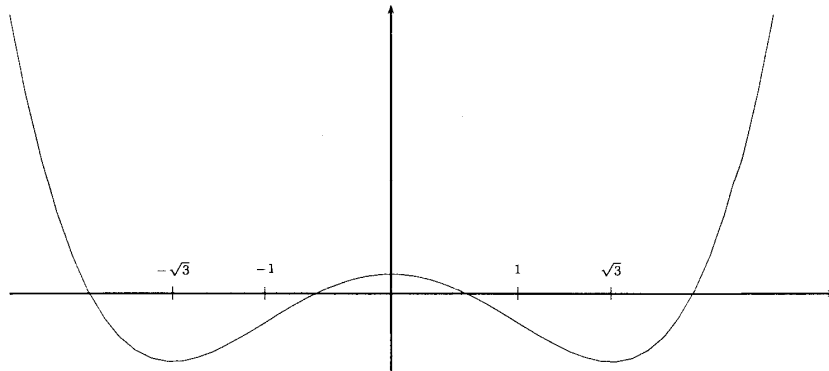
(b) $f' = 4x^3 - 12x = 4x(x^2 - 3)$.

$f' < 0$ for $x < -\sqrt{3}$ or in $(0, \sqrt{3})$; $f' > 0$ for $x > \sqrt{3}$ or in $(-\sqrt{3}, 0)$ (1pt).

Critical points are 0 (local max)(1pt) and $\pm\sqrt{3}$, local min(1pt). $f(\pm\sqrt{3}) = -7$

(c) $f'' = 12(x^2 - 1)$ (1/2pt). $f'' > 0$ for $|x| > 1$ (so concave up) and $f'' < 0$ on $(-1, 1)$, so concave down(1pt). Also ± 1 are points of inflection(1/2pt). $f'(1) = -3$.

(d) $f(-2) = -6, f(\pm\sqrt{3}) = -7, f(0) = 2, f(3) = 29$ (1pt). Absolute max at 3 , min at $\pm\sqrt{3}$ (1pt).



(e)