KUWAIT UNIVERSITY
Department of Mathematics and Computer Science

Math 101 — 10 May, 2009
Calculus (1) — Second Midterm — Time: 90 minutes

Answer all questions. Calculators and Mobile Phones are not allowed.

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1. (3pts) Let $f(t) = t^2 \sin t$. Find $f''(0)$.

2. (3pts) Suppose $y = f(x)$ is given by $x^2 + y^2 = 2y$. Find the critical numbers of f.

3. (3pts) A car starts at a point A and travels east at 80km/hr. At the same time another car starts at A and goes north at 60km/hr. At what rate is the distance between them changing one hour after the cars start?

4. (3pts) Find the linear approximation of $f(x) = \sqrt{2x+7}$ at $a = 1$.

5. (3pts) Let $f(x) = 6(2-x)^{4/3} + 5$.
   (a) Show that there is no $c \in (1,2)$ such that $f'(c) = 0$.
   (b) Does this contradict Rolle's Theorem for f on $[1,2]$? Explain.

6. (10pts) Let $f(x) = x^4 - 6x^2 + 2$.
   (a) Discuss the symmetry of the graph of $y = f(x)$.
   (b) Find the intervals on which f is increasing and the intervals on which f is decreasing. Find the local extrema of f, if any.
   (c) Find the intervals on which f is concave upwards and the intervals on which f is concave downwards. Find the inflection points of f, if any.
   (d) Find the absolute extrema of f on $[-2,3]$.
   (e) Sketch the graph of f.

Solutions

1. $f' = 2t\sin t + t^2\cos t$ (1pt), $f'' = 2\sin t + 4t\cos t - t^2\sin t$ (1pt), so $f''(0) = 0$(1pt).

2. $2x + 2yy' = 2y'$, so $y' = \frac{x}{1-y}$(1pt). Now, $y' = 0 \iff x = 0$ and does not exist $\iff y = 1, x = \pm 1$(1pt). Critical points are $0, \pm 1$(1pt).

3. Let x be distance travelled by first car, y distance travelled by second car after t hours. Distance between them is
   $D = \sqrt{x^2+y^2}$(1pt), $\frac{dD}{dt} = \frac{2xx'+2yy'}{2\sqrt{x^2+y^2}}$(1pt), so when $t = 1, x = 80, y = 60, x' = 80, y' = 60, dD/dt = 100$(1pt).

4. $f' = \frac{2}{2\sqrt{2x+7}}$(1pt), so $L = f(a) + f'(a)(x-a)$(1pt) $= 3 + \frac{1}{3}(x-1)$(1pt).

5. $f' = -6\frac{4}{3}(2-x)^{1/3}$(1pt) $= 0 \iff x = 2$, so no $c \in (1,2)$(1pt). Notice that $f(1) = 11 \neq f(2) = 5$, so hypotheses of Rolle not satisfied(1pt).

6. (a) (1pt) $f(-x) = f(x)$, f even, graph symmetric about y-axis.
   (b) $f' = 4x^3 - 12x = 4x(x^2-3)$.
   $f' < 0$ for $x < -\sqrt{3}$ or in $(0, \sqrt{3})$; $f' > 0$ for $x > \sqrt{3}$ or in $(-\sqrt{3}, 0)$(1pt).
   Critical points are 0 (local max)(1pt) and $\pm\sqrt{3}$, local min(1pt). $f(\pm\sqrt{3}) = -7$. $f(0) = 2$
   (c) $f'' = 12(x^2-1)$(1/2pt). $f'' > 0$ for $|x| > 1$ (so concave up) and $f'' < 0$ on $(-1, 1)$, so concave down(1pt). Also ± 1 are points of inflection(1/2pt). $f(\pm 1) = -3$.
   (d) $f(-2) = -6, f(\pm\sqrt{3}) = -7, f(0) = 2, f(3) = 29$(1pt). Absolute max at 3, min at $\pm\sqrt{3}$(1pt).
   (e)